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ABSTRACT

In this paper we introduce and investigate the notations of contra $\psi\alpha$ -continuous functions and almost contra $\psi\alpha$ -continuous functions in topological spaces. We discuss the relations between these contra continuities and other related contra continuities.

KEYWORDS: Contra continuous functions, almost contra continuous functions, $\psi\alpha$ -continuous functions, $\psi\alpha$ -closed sets and $\psi\alpha$ -open sets..

1. INTRODUCTION

Singal M.K and Singal A.R [12] introduced almost continuous functions in topological spaces. Levine [9] introduced the idea of continuous functions in topological spaces. Dontchev [3] introduced the notation of contra continuous functions in topological spaces. Jafari and Noiri [5] introduced and studied the new form of functions called contra α -continuous functions in topological spaces. Ekici [4] introduced the concept of almost contra continuous functions in topological spaces. Shakila and Balamani [11] introduced $\psi\alpha$ -continuous functions in topological spaces. Recently Karthika and Balamani [7] introduced totally $\psi\alpha$ -continuous functions and $\psi\alpha$ totally continuous functions in topological spaces.

The purpose of this paper is to introduce and study a new type of contra continuous functions namely contra $\psi\alpha$ -continuous functions and almost contra $\psi\alpha$ -continuous functions in topological spaces. Also we obtain the interrelations between these continuous functions.

2. PRELIMINARIES

Definition 2.1 Let (X, τ) be a topological space. A Subset A of a topological space (X, τ) is called

- 1) **Regular open** [13] if $A = \text{int}(\text{cl}(A))$.
- 2) **Semi-open** [8] if $A \subseteq \text{cl}(\text{int}(A))$.
- 3) **α -open** [10] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$.
- 4) **generalized closed** [9] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .
- 5) **Semi generalized closed** [2] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open in (X, τ) .
- 6) **ψ -closed** [14] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is sg-open in (X, τ) .
- 7) **$\psi\alpha$ -closed** [11] if $\psi\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .
- 8) **$\psi\alpha$ -clopen** [11] if it is both $\psi\alpha$ -open and $\psi\alpha$ -closed in (X, τ) .

Results 2.2

- Every **closed** (open) subset in (X, τ) is **$\psi\alpha$ -closed** ($\psi\alpha$ -open).
- Every **clopen** subset in (X, τ) is **$\psi\alpha$ -clopen**.
- Every **regular open** (regular closed) subset in (X, τ) is **open** (closed).
- Every **α -open** subset in (X, τ) is **$\psi\alpha$ -open**.

Definition 2.3 Let (X, τ) and (Y, σ) be two topological spaces. A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- 1) **Almost continuous** [12] if $f^{-1}(V)$ is closed in (X, τ) for every regular closed set V of (Y, σ) .

- 2) **Continuous** [9] if $f^{-1}(V)$ is closed in (X, τ) for every closed set V of (Y, σ) .
- 3) **Completely continuous** [1] if $f^{-1}(V)$ is regular open in (X, τ) for every open set V of (Y, σ) .
- 4) **Totally Continuous** [6] if $f^{-1}(V)$ is clopen in (X, τ) for every open set V of (Y, σ) .
- 5) **Almost contra continuous** [4] if $f^{-1}(V)$ is closed in (X, τ) for every regular open set V of (Y, σ) .
- 6) **Contra continuous** [3] if $f^{-1}(V)$ is closed in (X, τ) for every open set V of (Y, σ) .
- 7) **Contra α -continuous** [5] if $f^{-1}(V)$ is α -closed in (X, τ) for every open set V of (Y, σ) .
- 8) **$\psi\alpha$ -Continuous** [11] if $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) for every closed set V of (Y, σ) .
- 9) **Totally $\psi\alpha$ -continuous** [7] if $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) for every open set V of (Y, σ) .
- 10) **$\psi\alpha$ -totally continuous** [7] if $f^{-1}(V)$ is clopen in (X, τ) for every $\psi\alpha$ -open set V of (Y, σ) .

3. CONTRA $\psi\alpha$ -CONTINUOUS FUNCTIONS

Definition 3.1 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **contra $\psi\alpha$ -continuous** if $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) for every closed set V of (Y, σ) .

Example 3.2 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is contra $\psi\alpha$ -continuous.

Theorem 3.3 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is contra $\psi\alpha$ -continuous if and only if $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) for every open set V of (Y, σ) .

Proof: (Necessity) Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be contra $\psi\alpha$ -continuous and V be any open set in (Y, σ) . Then $Y - V$ is closed in (Y, σ) . Since f is contra $\psi\alpha$ -continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) .

(Sufficiency): Let U be any closed set in (Y, σ) . Then $Y - U$ is open in (Y, σ) . By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is $\psi\alpha$ -open in (X, τ) which implies that $f^{-1}(U)$ is $\psi\alpha$ -closed in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Proposition 3.4 Every contra continuous function is a contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any open set in (Y, σ) . Since f is contra continuous, $f^{-1}(V)$ is closed in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Example 3.5 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is contra $\psi\alpha$ -continuous but not contra continuous, since for the open set $\{a\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{b\}$ is $\psi\alpha$ -closed but not closed in (X, τ) .

Proposition 3.6 Every contra α -continuous function is a contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any closed set in (Y, σ) . Since f is contra α -continuous, $f^{-1}(V)$ is α -open in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Example 3.7 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is contra $\psi\alpha$ -continuous but not contra α -continuous, since for the closed set $\{b, c\}$ in (Y, σ) , $f^{-1}(\{b, c\}) = \{a, c\}$ is $\psi\alpha$ -open but not α -open in (X, τ) .

Proposition 3.8 Every totally continuous function is a contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any open set in (Y, σ) . Since f is totally continuous, $f^{-1}(V)$ is clopen in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Example 3.9 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = b$, $f(b) = a$, $f(c) = c$. Then f is contra $\psi\alpha$ -continuous but not totally continuous, since for the open set $\{a\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{b\}$ is $\psi\alpha$ -closed but not clopen in (X, τ) .



Proposition 3.10 Every totally $\psi\alpha$ -continuous function is a contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any closed set in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Example 3.11 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is contra $\psi\alpha$ -continuous but not totally $\psi\alpha$ -continuous, since for the closed set $\{c\}$ in (Y, σ) , $f^{-1}(\{c\}) = \{a\}$ is $\psi\alpha$ -open but not $\psi\alpha$ -closed in (X, τ) .

Proposition 3.12 Every $\psi\alpha$ -totally continuous function is a contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any open set in (Y, σ) . By result 2.2, V is $\psi\alpha$ -open in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X, τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) . Hence f is contra $\psi\alpha$ -continuous.

Example 3.13 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is contra $\psi\alpha$ -continuous but not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{a, b\}$ in (Y, σ) , $f^{-1}(\{a, b\}) = \{a, c\}$ is not clopen in (X, τ) .

Remark 3.14 Contra $\psi\alpha$ -continuous function is independent from $\psi\alpha$ -continuous function as seen from the following examples.

Example 3.15 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a) = c$, $f(b) = b$, $f(c) = a$. Then f is contra $\psi\alpha$ -continuous but not $\psi\alpha$ -continuous, since for the closed set $\{c\}$ in (Y, σ) , $f^{-1}(\{c\}) = \{a\}$ is $\psi\alpha$ -open but not $\psi\alpha$ -closed in (X, τ) .

Example 3.16 Let $X = Y = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is $\psi\alpha$ -continuous but not contra $\psi\alpha$ -continuous, since for the closed set $\{c\}$ in (Y, σ) , $f^{-1}(\{c\}) = \{c\}$ is $\psi\alpha$ -closed but not $\psi\alpha$ -open in (X, τ) .

Proposition 3.17 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any closed set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ) . Since f is contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.18 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a totally $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any closed set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.19 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any closed set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ) . Since f is totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is contra $\psi\alpha$ -continuous.

Proposition 3.20 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a $\psi\alpha$ -totally continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.



Proof: Let V be any closed set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is closed in (Y, σ) . By result 2.2, $g^{-1}(V)$ is $\psi\alpha$ -closed in (Y, σ) . Since f is $\psi\alpha$ -totally continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -clopen in (X, τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.21 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a totally continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any closed set in (Z, η) . Since g is totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is closed in (Y, σ) . Since f is contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.22 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a $\psi\alpha$ -totally continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any closed set in (Z, η) . By result 2.2 V is $\psi\alpha$ -closed in (Z, η) . Since g is $\psi\alpha$ -totally continuous, $g^{-1}(V)$ is clopen in (Y, σ) which implies that $g^{-1}(V)$ is closed in (Y, σ) . Since f is contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -open in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.23 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a totally $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a completely continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y, σ) . By result 2.2, $g^{-1}(V)$ is open in (Y, σ) . Since f is totally $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -clopen in (X, τ) which implies that $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Proposition 3.24 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is continuous, $g^{-1}(V)$ is open in (Y, σ) . Since f is contra continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed in (X, τ) . By result 2.2, $(g \circ f)^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

Remark 3.25 The composition of two contra $\psi\alpha$ -continuous functions need not be a contra $\psi\alpha$ -continuous function as seen from the following example.

Example 3.26 Let $X=Y=Z=\{a,b,c\}$, $\tau=\{\emptyset, \{a\}, \{a,b\}, X\}$ and $\sigma=\{\emptyset, \{a,b\}, Y, \eta=\{\emptyset, \{a\}, Z\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a)=c$, $f(b)=b$, $f(c)=a$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a function defined by $g(a)=c$, $g(b)=b$, $g(c)=a$. Then the functions f and g are contra $\psi\alpha$ -continuous, but their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is not contra $\psi\alpha$ -continuous, since for the closed set $\{b,c\}$ in (Z, η) , $(g \circ f)^{-1}(\{b,c\})=\{b,c\}$ is not $\psi\alpha$ -open in (X, τ) .

4. ALMOST CONTRA $\psi\alpha$ -CONTINUOUS FUNCTIONS

Definition 4.1 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called **almost contra $\psi\alpha$ -continuous** if $f^{-1}(V)$ is $\psi\alpha$ -closed in (X, τ) for every regular open set V of (Y, σ) .

Example 4.2 Let $X=Y=\{a,b,c\}$, $\tau=\{\emptyset, \{a\}, \{a,b\}, X\}$ and $\sigma=\{\emptyset, \{a\}, \{b\}, \{a,b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a function defined by $f(a)=c$, $f(b)=b$, $f(c)=a$. Then f is an almost contra $\psi\alpha$ -continuous function.

Theorem 4.3 A function $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost contra $\psi\alpha$ -continuous if and only if the inverse image of every regular open subset of (Y, σ) is $\psi\alpha$ -closed in (X, τ) .

Proof:(Necessity) Let $f:(X,\tau) \rightarrow(Y,\sigma)$ be almost contra $\psi\alpha$ -continuous. Let V be any regular open set in (Y,σ) . Then $Y-V$ is regular closed in (Y,σ) . Since f is almost contra $\psi\alpha$ -continuous, $f^{-1}(Y-V) = X - f^{-1}(V)$ is $\psi\alpha$ -open in (X,τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) .

(Sufficiency): Let U be any regular closed set in (Y,σ) . Then $Y-U$ is regular open in (Y,σ) . By assumption, $f^{-1}(Y-U) = X - f^{-1}(U)$ is $\psi\alpha$ -closed in (X,τ) which implies that $f^{-1}(U)$ is $\psi\alpha$ -open in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Proposition 4.4 Every contra $\psi\alpha$ -continuous function is almost contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ) . By result 2.2, V is open in (Y,σ) . Since f is contra $\psi\alpha$ -continuous, $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Example 4.5 Let $X = Y = \{a,b,c\}, \tau = \{\phi, \{a\}, \{b\}, \{a,b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, Y\}$. Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be the identity function. Then f is almost contra $\psi\alpha$ -continuous but not contra $\psi\alpha$ -continuous, since for the open set $\{a,b\}$ in (Y,σ) , $f^{-1}(\{a,b\}) = \{a,b\}$ is not $\psi\alpha$ -closed in (X,τ) .

Proposition 4.6 Every contra continuous function is almost contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ) . By result 2.2, V is open in (Y,σ) . Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Example 4.7 Let $X=Y=\{a,b,c\}, \tau = \{\phi, \{a\}, \{a,b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a,b\}, Y\}$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a function defined by $f(a)=c, f(b)=b, f(c)=a$. Then f is almost contra $\psi\alpha$ -continuous but not contra continuous, since for the open set $\{b\}$ in (Y,σ) , $f^{-1}(\{b\}) = \{b\}$ is not closed in (X,τ) .

Proposition 4.8 Every totally $\psi\alpha$ -continuous function is almost contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ) . By result 2.2, V is open in (Y,σ) . Since f is totally $\psi\alpha$ -continuous, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X,τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Example 4.9 Let $X = Y = \{a,b,c\}, \tau = \{\phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a,b\}, Y\}$. Let $f: (X,\tau) \rightarrow (Y,\sigma)$ be a function defined by $f(a)=c, f(b)=b, f(c)=a$. Then f is almost contra $\psi\alpha$ -continuous but not totally $\psi\alpha$ -continuous, since for the open set $\{a\}$ in (Y,σ) , $f^{-1}(\{a\}) = \{c\}$ is not $\psi\alpha$ -clopen in (X,τ) .

Proposition 4.10 Every $\psi\alpha$ -totally continuous function is almost contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ) . By result 2.2, V is open in (Y,σ) which implies that V is $\psi\alpha$ -open in (Y,σ) . Since f is $\psi\alpha$ -totally continuous, $f^{-1}(V)$ is clopen in (X,τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X,τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Example 4.11 Let $X = Y = \{a,b,c\}, \tau = \{\phi, \{a\}, \{a,b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a,b\}, Y\}$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be a function defined by $f(a)=c, f(b)=b, f(c)=a$. Then f is almost contra $\psi\alpha$ -continuous but not $\psi\alpha$ -totally continuous, since for the $\psi\alpha$ -open set $\{a,c\}$ in (Y,σ) , $f^{-1}(\{a,c\}) = \{a,c\}$ is not clopen in (X,τ) .

Proposition 4.12 Every totally continuous function is almost contra $\psi\alpha$ -continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ) . By result 2.2, V is open in (Y,σ) . Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ) . By result 2.2, $f^{-1}(V)$ is $\psi\alpha$ -clopen in (X,τ) which implies that $f^{-1}(V)$ is $\psi\alpha$ -closed in (X,τ) . Hence f is almost contra $\psi\alpha$ -continuous.

Example 4.13 Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be the identity function. Then f is almost contra $\psi\alpha$ -continuous but not totally continuous, since for the open set $\{a\}$ in (Y, σ) , $f^{-1}(\{a\}) = \{a\}$ is not clopen in (X, τ) .

Proposition 4.14 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a completely continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is almost contra $\psi\alpha$ -continuous function.

Proof: Let V be any regular open set in (Z, η) . By result 2.2, V is open in (Z, η) . Since g is completely continuous function, $g^{-1}(V)$ is regular open in (Y, σ) . Since f is almost contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -closed in (X, τ) . Hence $g \circ f$ is almost contra $\psi\alpha$ -continuous function.

Proposition 4.15 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is almost continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is almost contra $\psi\alpha$ -continuous function.

Proof: Let V be any regular open set in (Z, η) . Since g is almost continuous, $g^{-1}(V)$ is open in (Y, σ) . Since f is contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -closed in (X, τ) . Hence $g \circ f$ is almost contra $\psi\alpha$ -continuous function.

Proposition 4.16 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is almost contra $\psi\alpha$ -continuous function and $g: (Y, \sigma) \rightarrow (Z, \eta)$ is a completely continuous function, then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a contra $\psi\alpha$ -continuous function.

Proof: Let V be any open set in (Z, η) . Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y, σ) . Since f is almost contra $\psi\alpha$ -continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\psi\alpha$ -closed in (X, τ) . Hence $g \circ f$ is a contra $\psi\alpha$ -continuous function.

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